Department: Elec. Power & Comm.	Pyramids higher Institute P.H.I. For Engineering And Technology معهد الأهرامات العالى للهندسة و التكنولوجيا	Final Exam
Level: 3		Course Title : Mathematics 5
Examiner: Dr. Mohamed Eid		Course Code: BAS 216
Time allowed: 3 hours		Date: January 21, 2018

The Exam consists of one page   Answer all questions   No. of questions 5   Total Mark 70

Question 1 Complete the following :

(a) A square matrix A is called non singular if ……………………………………..

(b) The eigenvalues of a symmetric matrix of real numbers are ………………………..

(c) If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are all eigenvalues of a matrix A, then trace A is…………………..…

(d) The linear system AX = 0 has zero solution if ………………………………………

(e) If f(z) = u + iv is analytic complex function, then u and v are…………………….

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Question 2

(a) Find the eigenvalues and the eigenvectors of the matrix : $A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$.

   Also, Find the trace of A, Write the Hamilton equation and the diagonal form.

   Show that the eigenvectors are orthogonal.

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(b) Solve the linear systems :

   (i) $x - 2y + 2z = 3$, $2x + y - z = 2$, $3x - y + z = 6$

   (ii) $x - 2y + 2z = 2$, $2x - z = 3$, $3x - y + 2z = 7$

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Question 3

(a) Find the integrals : (i) $\int_1^2 \dfrac{x}{\sqrt{x^3 - 1}}\,dx$ (ii) $\int_1^\infty \dfrac{1}{\sqrt{1 + x^6}}\,dx$

(b) Find the curves : $y = a + b\ln x$, $y = a + bx + cx^2$ that fit the data :

   (2, 3), (4, 4), (6, 7), (8, 9), (10, 12)

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Question 4

(a) Find u and v of the function : $f(z) = \cos 2z$ and show that they are harmonic.

(b) Show that the image of the circle $|z| = 3$ is the line $u = \ln 3$ under the function $f(z) = \ln z$.

(c) Determine and sketch the image of the region $G : 0 \le x \le \dfrac{\pi}{2}, \ 0 \le y \le 4$ under the function $f(z) = \sin z$.

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Question 5

(a) If C is $|z| = 3$, find (i) $\oint_C \dfrac{\sin 2z}{z^2 + 16}\,dz$ (ii) $\oint_C \dfrac{e^{2z}}{z^2}\,dz$ (iii) $\oint_C \dfrac{(z + 4)^2}{z^2 - 4}\,dz$

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(b) Find the integral $\displaystyle\int_{-\infty}^{\infty} \dfrac{\cos x}{x^2 + 4}\,dx$

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(c) Find c such that the function f(x) is p.d.f : $f(x) = \begin{cases} c(2x - x^2), & 0 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}$

   Also, find : (i) F(x)   (ii) F(0), $F\left(\dfrac{1}{2}\right)$, F(2), $P(\dfrac{1}{2} < x \le 1)$

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Good Luck   *Dr. Mohamed Eid*